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Effective thermal conductivity of frost during the crystal growth period

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Abstract

An effective frost thermal conductivity for the crystal growth period of frost formation was developed based on a simple model consisting of cylindrical frost columns surrounded by moist air. Being closely related to the frost layer density, the thermal conductivity of frost layer is found to be affected by the vapor diffusion inside the frost layer as well. The density of frost columns, on the other hand, depends on the sublimation temperature of ice crystals and the humidity ratio of air, therefore, the whole process of frost formation and the thermal conductivity of frost layer, in particular, is shown to depend on the controlling temperatures, i.e. plate temperature, air temperature, and air humidity ratio. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Frost formation is an important consideration in cryogenics, refrigeration, air conditioning, and aerospace industry. Nocturnal frost on aircraft wings is known to cause aerodynamic penalties of lift and drag during take-off. It is required that the frost developed on the outer surface of the transport planes be removed prior to take-off. This sometimes requires a costly operation in which an expensive petroleum solvent is used. Frost formation on the external tank of a space shuttle and space vehicles alike is particularly important because the built up frost may shed and damage the surface tiles of the vehicle during take-off.

Frost formation encountered in the field of refrigeration and air-conditioning, has a significant adverse effect upon the heat transfer and pressure drop. For instance, the frost formation on heat exchanger surfaces can be extremely detrimental to their efficient operation since the frost will act as a thermal insulator, thus reducing the ability of the surface to transfer heat. Also, accumulations of the frost often become thick enough to restrict and block air flow. There is a need for a fundamental understanding of the nature of frost formation including the vapor-condensation process to assist in predicting rate of frost formation. A number of investigators have made both experimental and theoretical studies of the heat and mass transfer in frost in an effort to obtain both the rate of heat transfer and the thermal conductivity. Discrepancies appear primarily as a result of the lack of information on the dependence of the frost structure and, therefore, the frost properties on the experimental conditions.

The thermal conductivity of frost depends on a variety of parameters, thus making it almost impossible to select an accurate value for calculations. The temperature at which frost crystals form would influence the type of crystalline formation and therefore directly affect the thermal conductivity. The thermal conductivity of frost is affected by the density; however, it must be a function of some other factors as well. It can be shown that two frost layers with identical densities might have different thermal conductivities. One of the reasons is that sublayers of various densities exist within the frost layer under certain environmental conditions. The shape and orientation of the ice crys-

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Nomenclature

A_c	area of frost columns per unit area of frost layer
A_v	area of the void parts per unit area of the frost layer
D	diffusivity of water vapor in air (m^2/s)
D_H	hydraulic diameter (m)
h_c	heat transfer coefficient (W/m^2 K)
h_g	enthalpy of saturated water vapor (J/kg)
h_{ig}	latent heat of sublimation (J/kg)
k_a	thermal conductivity of moist air (W/m K)
k_f	frost layer thermal conductivity (W/m K)
k_{fc}	frost column thermal conductivity (W/m K)
k_{eff}	effective frost thermal conductivity (W/m K)
L_f	frost layer thickness (m)
$\dot{m}$	total mass flux per unit area (kg/m^2 s)
$\dot{m}_d$	mass diffusion rate of water vapor into the frost layer per unit area of frost layer (kg/m^2 s)
$\dot{m}_s$	sublimation rate of water vapor inside the frost layer per unit volume of frost layer (kg/m^3 s)
P_g	partial pressure of water vapor (kPa)
P_0	partial pressure of water vapor at $T_0 = 273.15$ K (kPa)
Pr	Prandtl number
q_a	conduction heat flux through the voids of frost layer (W/m^2)
q_c	conduction heat flux through the frost columns (W/m^2)
Re	Reynolds number
Sc	Schmidt number
T_a	air temperature (K)
T_p	plate temperature (K)
T_s	frost surface temperature (K)
u_i	internal energy of ice (J/kg)
y	distance from the plate surface (m)
β	volumetric ratio of frost columns
β_0	initial volumetric ratio of frost columns
ρ_a	density of air (kg/m^3)
ρ_c	sublimation density of ice crystal (kg/m^3)
ρ_f	frost density (kg/m^3)
ω_a	humidity ratio of moist air
ω_s	humidity ratio of air at frost surface

tals within the frost layer structure affect the thermal conductivity as well. For a given density, if the crystals formed an assembly of thin columns, all of which were oriented in planes parallel to the direction of heat flow, frost layer would have its highest thermal conductivity. However, for the same given density, a low thermal conductivity would occur if the ice crystals oriented themselves in planes perpendicular to the direction of the heat flow.

Yonko and Sepsy [1] developed a theoretical model for the frost thermal conductivity by postulating an effective air conductivity and then proceeding to treat the frost as a cubical lattice of uniform spherical ice particles in moist air. They correlated their experimental data of frost thermal conductivity as function of density, however also found a considerable scatter. They concluded that the conductivity of frost is a function of factors other than the frost density. This means that the true relationship between the thermal conductivity and the density would be given by an area rather than a line on a thermal conductivity and density plot.

Trammell et al. [2] performed experimental tests and numerical calculations to determine values of the thermal conductivity, convective heat transfer coefficient, and overall heat transfer coefficient of frost forming on a flat plate. They found that increases in the air velocity and air specific humidity resulted in an increase in the frost thermal conductivity. In addition, experimental to theoretical data comparisons were made for several parameters. They have studied the effect of velocity, humidity and temperature on frost formation and have discussed the density profile in a frost layer.

Brian et al. [3] performed experimental measurements to determine the variations of density and thermal conductivity of frosts forming on cold surfaces at cryogenic temperatures. Later, an empirical expression for the thermal conductivity of frost, based on experimental data, was developed by Brian et al. [4] as a function of both the temperature and the density. If Brian et al.'s empirical frost thermal conductivities as a function of temperature are calculated for high densities then they do not approach the thermal conductivity of ice, as required. In addition, they stated that their model would not be expected to apply to the early stages of frost formation.

Biguria and Wenzel [5] have compiled a list of several possible structural models for the frost (ice and air in series, ice and air in parallel, ice in continuous phase with air pores arranged in a simple cubic lattice, uniform spheres of ice dispersed in air, random mixture of ice and air, cubic lattice of uniform ice particles in air) to come up with different frost thermal conductivity predictions. They concluded that at very low densities the theoretical models could not correlate frost density and thermal conductivity. Therefore they studied the main variables including free stream velocity, free stream humidity, boundary layer regime, plate temperature, and time.

Yamakawa et al. [6] pointed out that the effective thermal conductivity of the frost layer is influenced by the density; however, the relationship was not obvious and mass transfer caused by moisture diffusion must be considered in determining the thermal conductivity of the frost layer.

In another approach, White and Cremers [7] formulated the effective thermal conductivity as a linear function of frost density and an arbitrary function of frost interior temperature.

Dietenberger [8] has developed a method of calculating the frost thermal conductivity based on both theory and experimental data. His model, which is mostly theoretical and partly empirical, postulates a complicated frost structure which accounts for vapor diffusion, geometrical shapes of ice dendrites and for frost aging. It is basically a random mixture model with ice cylinders and ice spheres immersed in moist air as the predominating structure at low frost densities, and with ice planes in moist air and moist air bubbles in ice at high densities. The proportions of these various structures are then calculated using experimental data and a generalized correlation of frost thermal conductivity is presented which depends on frost temperature and density. The average frost thermal conductivity results in his model at high frost densities are unrealistically high.

Tokura et al. [9] found that the parameters affecting frost density are also the major factors in the thermal conductivity of a frost layer. More recent investigators such as Sami and Duong [10], Sherif et al. [11], Tao et al. [12] and Ismail et al. [13] used Yonko and Sepsy's [1] model of thermal conductivity as function of density as a sole parameter, without questioning whether the frost density can be considered as a sole parameter. Fukada et al. [14] studied frost thermal conductivity under varying gaseous atmospheres and found that the scatter on a thermal conductivity versus density plot was so large that they were unable to correlate them. Le Gall et al. [15] used the model of Auracher [16] which gives thermal conductivity as a function of both the density and the frost temperature. Lee et al. [17] used a new thermal conductivity model that they developed earlier. Their model being a function of only density behaves poorly for the early stages of frost formation during which the frost density is low. Fig. 1 shows a comparison of some of the earlier frost thermal conductivity models including models from snow literature [19–23]. The large discrepancy at all stages of frost formation is obvious.

An analytical study on the frost formation was made by Şahin [24] to clarify the fundamental nature

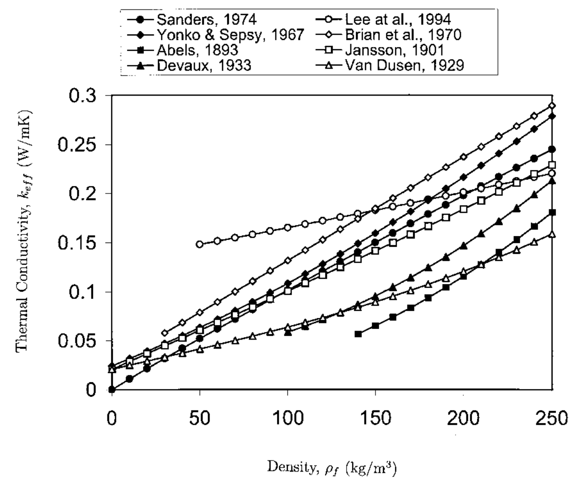

Fig. 1. A comparison of some of the earlier frost thermal conductivity models.

of the early stage (crystal growth period) of frost formation phenomena. A suitable model was developed by using the principles of crystallization and nucleation theory. Ice crystal density variation with temperature reported by cloud physicists is used in the model to predict the density variation of frost during the crystal growth period. The temperature variation in the frost layer is formulated and vapor diffusion through the frost layer is taken into consideration.

Consequently, in the present work, the effective thermal conductivity of frost layer during the crystal growth period is studied. Using the model developed by Şahin [24], it is shown that the effective thermal conductivity of frost layer is not only a function of frost density, but depends on the diffusion of water vapor and sublimation activity which take place in the frost layer as well.

In the next section, a brief introduction to ice crystal structure under different ambient conditions is made and in the subsequent section, a model of frost thermal conductivity for crystal growth period of frost formation that is based on the ice crystal structure is given.

2. Ice crystal structure

Two general classifications of crystals may be identified for ice. In their simplest form, these are *columns* and *plates*. Both types of crystals may exhibit a skeletal or incomplete structure. Columns with incomplete basal faces are termed *hollow columns*, *sheaths* or in the extreme case, *needles*. Plates which form platelike extensions of the prism faces are termed *sector plates*, or dendrites (Fig. 2).

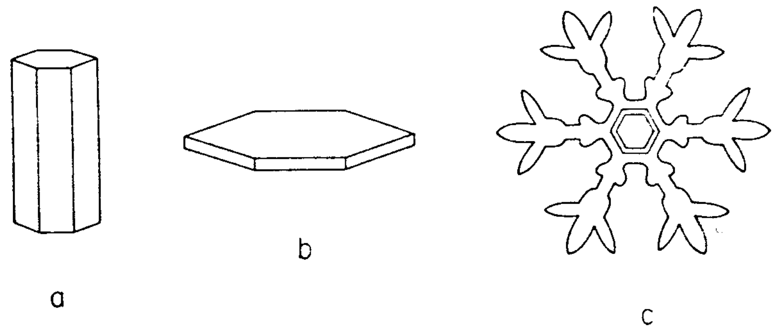

Fig. 2. Main type of ice crystal structures: (a) column, (b) plate, (c) dendrite.

Many workers have noted that the habit (shape) of ice crystals exhibits a striking dependence on temperature and, to a lesser degree, on supersaturation. Mason and Hallet [25] were able to grow crystals over the temperature range 273–223 K and under supersaturations ranging from a few per cent to about 300% using a thin nylon or glass fibre running vertically through the center of a water vapor diffusion chamber. The crystal habit varied along the length of the fibre as shown in Table 1.

Transitions from plate growth to prisms growth, back to plates and then to prisms again may be noted as the temperature decreases. The classification of laboratory-produced crystals according to the temperature of formation bears a marked similarity to that of natural snow crystals, showing that it is possible to simulate quite well the early stages of growth of snow crystals in the laboratory and, at the same time, determine the transition temperatures for the different crystal forms more precisely than can be done in the atmosphere. The most striking feature of crystal habit is the remarkable sequence of habit, plates–prisms–plates (and stars)–prisms, which occurs as the temperature is lowered from 273 to 248 K.

Table 1
Variation of crystal habit with temperature

Temperature range (K)	Form of ice crystal
273–270	Thin hexagonal plates
270–268	Needles
268–265	Hollow prisms
265–261	Hexagonal plates
261–257	Dentritic crystals
257–248	Plates
248–223	Hollow prisms

Very large variations of supersaturation do not change the basic crystal habit between prism and plate-like growth although, of course, the growth rates are profoundly affected. On the other hand, the supersaturation appears to govern the development of various secondary features such as the needle-like extensions of hollow prism, the growth of spikes and sectors at the corners of hexagonal plates, and the fern-like development of the star-shaped crystals, all of which occur only if the supersaturation exceeds values which correspond roughly to saturation relative to liquid water.

Whenever a crystal was transferred to a new environment, the continued growth assumed a new habit characteristic of the new conditions. Thus, when needles grown at temperatures between 268 and 270 K are suddenly moved up in the chamber, to about 271 K, plates develop on their ends, and when similar needles are lowered to about 259 K, they give way to star-shaped crystals. Such radical changes in crystal shape could not be produced by varying the supersaturation at constant temperature, but in some cases were produced by only a degree or two change in temperature at constant supersaturation [25].

The sublimation density of ice crystals as a function of crystal temperature, based on the experimental studies of Fukuta [26] and Miller and Young [27] is shown in Fig. 3. The least squares approximation of the experimental data that is used for the crystal growth model given in the following section is

$$\rho_c = \begin{cases} -10429.56 + 41.574T & 255.15\ \mathrm{K} < T < 273.15\ \mathrm{K} \\ 180 & T < 255.15\ \mathrm{K} \end{cases} \tag{1}$$

where T is the crystal formation temperature in K and ρ_c is in kg/m^3.

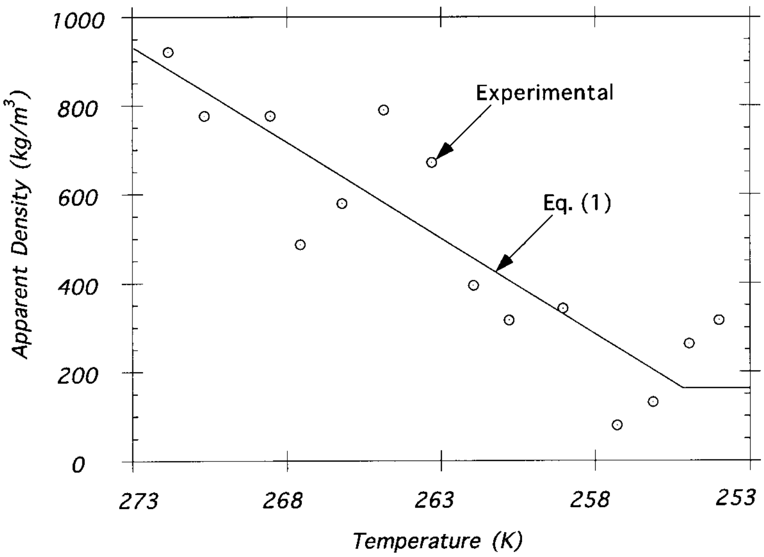

Fig. 3. The sublimation density of ice crystals as a function of crystal temperature.

3. Model for crystal growth period

A simple model, shown in Fig. 4, has been selected for this work in which the frost layer is assumed to consist of several frost columns.

The following assumptions are made for this model:

- Frost columns consist of ice crystals.
- Volumetric ratio of frost columns, β, which is defined as the ratio of the total volume of the frost columns to the total volume of frost on a unit area of plate surface, is a function of time and has an initial suggested value, β_0.
- The temperature of the frost column at a cross-section y is the same as that of the void portion at this position and varies with y.
- The humidity of the air at any cross-section corresponds to saturation humidity at that temperature.
- The conductivity of frost columns, k_{fc}, is a function of the frost column density only.

The process of adhesion and growth of frost on the heat transfer plate is a very slow but unsteady process. However, because this process is very slow, it is treated in a quasi-steady state manner.

3.1. Analysis of crystal growth model

Frost thermal conductivity is one of the critical properties that affect frost growth and heat transfer and therefore, need to be studied in detail. Since frost nucleation and growth during the early stage of formation has different and unique structures, the correlations developed for relatively thick frost layers are not suitable for this study. The large discrepancy among these models also indicates that there is indeed a need for a closer look at the thermal conductivity of frost during the crystal growth period. With these considerations, an attempt to develop a theoretical model of frost thermal conductivity that could be suitable for the early stage of frost formation is presented in the following.

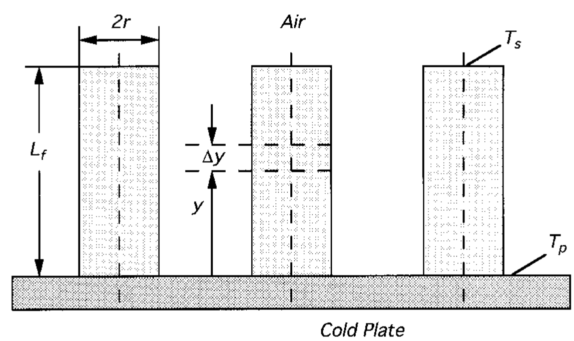

Fig. 4. Frost model consisting of cylindrical frost columns.

The conservation of mass on the differential control volume at location y in the frost layer as shown in Fig. 5 is

$$\dot{m}_s = -\frac{d\dot{m}_d}{dy} \tag{2}$$

where $\dot{m}_s$ (kg/m^3 s) is the rate of sublimation of water vapor inside the differential control volume. An energy balance on the same differential control volume as shown above in Fig. 6 yields,

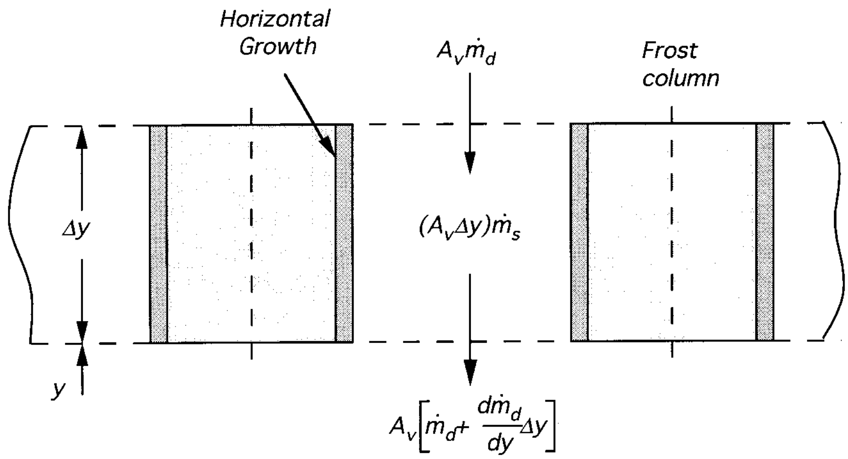

Fig. 5. Conservation of mass inside the frost layer.

$$A_v \frac{d(\dot{m}_d h_g)}{dy} - A_v \dot{m}_s u_i - A_c \frac{dq_c}{dy} - A_v \frac{dq_a}{dy} = 0 \tag{3}$$

where A_c and A_v are the total cross-sectional area of frost columns and the total cross-sectional area of void portions of the frost per unit area of frost, respectively.

It should be noted that $(h_g - u_i) \approx h_{ig}$ is relatively constant within the range of temperature of most practical frosting conditions. Thus, combining Eqs. (2) and (3) the energy balance equation becomes

$$-A_v \dot{m}_d \frac{dh_g}{dy} - A_v h_{ig} \frac{d\dot{m}_d}{dy} - A_c \frac{dq_c}{dy} - A_v \frac{dq_a}{dy} = 0. \tag{4}$$

Since the temperature variation inside the frost layer is not large [12], the first term in Eq. (4) is negligible in comparison with the other terms. Thus, neglecting the first term in Eq. (4), the energy balance is obtained as

$$-A_v h_{ig} \frac{d\dot{m}_d}{dy} - A_c \frac{dq_c}{dy} - A_v \frac{dq_a}{dy} = 0. \tag{5}$$

The vapor mass diffusion rate inside the frost layer is given by

$$\dot{m}_d = -D\rho_a \frac{d\omega_s}{dy} = -D\rho_a \frac{d\omega_s}{dP_g} \frac{dP_g}{dT} \frac{dT}{dy}. \tag{6}$$

Thus, the energy balance can be written as

$$\frac{d}{dy}\left\{\left[(1-\beta)h_{ig} D\rho_a \frac{d\omega_s}{dP_g}\frac{dP_g}{dT} + \beta k_{fc} + (1-\beta)k_a\right]\frac{dT}{dy}\right\} = 0 \tag{7}$$

where the volumetric ratio of frost columns is defined as

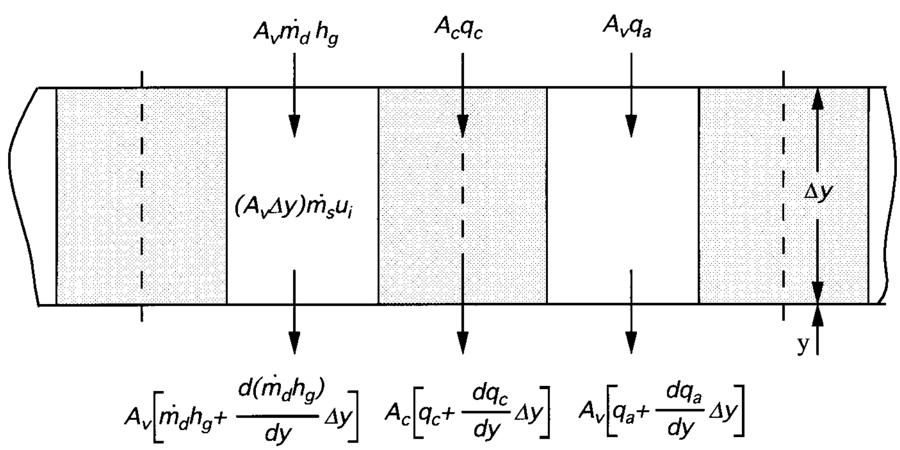

Fig. 6. Energy balance inside the frost layer.

$$\beta = \frac{A_c}{A_c + A_v}.$$

The term in the square bracket in Eq. (7) is the local frost thermal conductivity

$$k_f = (1-\beta)h_{ig}D\rho_a \frac{d\omega_s}{dP_g}\frac{dP_g}{dT} + \beta k_{fc} + (1-\beta)k_a. \tag{8}$$

Under the frosting conditions

$$\rho_a \approx \frac{P_{atm}}{RT},$$

$$\omega_s \approx 0.622 \times 10^{-2} P_g, \tag{9}$$

where P_g is in kPa and

$$P_g = P_0 \exp\left[\frac{h_{ig}}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \tag{10}$$

where P_0 is the partial pressure of water vapor in kPa at $T_0 = 273.15$ K.

On the other hand, the diffusivity of water vapor in air is given by Pruppacher and Klett [28] as

$$D = 2.11 \times 10^{-5}\left(\frac{T}{T_0}\right)^{1.94}$$

with T in K and D in m^2/s. Thermal conductivity of the frost columns can be approximated by one of the many empirical relations found in the literature such as Sanders equation [18]

$$k_{fc} = 1.202 \times 10^{-3}(\rho_c)^{0.963}$$

and the thermal conductivity in the air side can be approximated by [28]

$$k_a \approx k_{air} = (1.0465 + 0.017T) \times 10^{-5}$$

with T in K, ρ in kg/m^3 and k in W/m K.

Thus, the frost thermal conductivity can be related to temperature as a sole parameter

$$\begin{aligned} k_f(T) = {} & 0.131 \times 10^{-6}(1-\beta) \\ & \times \frac{h_{ig}P_{atm}P_0}{T_0^{1.94}R^2T^{1.06}}\exp\left[\frac{h_{ig}}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \\ & + 1.202 \times 10^{-3}\beta(\rho_c)^{0.963} + (1-\beta)(1.0465 \\ & + 0.017T) \times 10^{-5}. \end{aligned} \tag{11}$$

The boundary condition on the surface of the frost layer must satisfy the equation

$$k_f \frac{dT}{dy}\Big|_{T=T_s} = h_c(T_a - T_s) + h_{ig}\left[\dot{m} - (1-\beta)\dot{m}_d|_{T=T_s}\right] \tag{12}$$

where the average heat transfer coefficient is given by [29]

$$h_c = 0.023\frac{k_a}{D_H}Re^{0.8}Pr^{0.3} \tag{13}$$

in W/m^2 K and the average diffusion rate of water mass through the moist air boundary layer is given by [29]

$$\dot{m} = 0.023\frac{D}{D_H}Re^{0.8}Sc^{0.3}(\omega_a - \omega_s)\rho_a \tag{14}$$

in kg/m^2 s, where Re is the Reynolds number, Pr is the Prandtl number and Sc is the Schmidt number. It is known that the frost layer influences the heat transfer coefficient. Therefore, Eqs. (13) and (14) which are valid for plane wall without frost should be regarded as approximations.

3.2. Effective frost thermal conductivity

The local thermal conductivity of the frost layer given by Eq. (8) consists of three terms. The first term stands for the effect of the diffusion and sublimation of water vapor in the frost layer to the frost thermal conductivity. This term clearly is a function of temperature, therefore, the contribution of this term to the frost thermal conductivity varies with the location in the frost layer. The second and third terms in Eq. (8) are the thermal conductivity contributions of the frost column and the moist air around the frost columns, respectively. These two terms are also closely related to frost temperature and vary along the thickness of the frost layer. Thus an effective frost thermal conductivity can be defined as

$$\frac{1}{k_{eff}} \equiv \frac{1}{L_f}\int_0^{L_f}\frac{1}{k_f(T)}\,dy \tag{15}$$

where $k_f(T)$ is given explicitly in Eq. (11).

Effective frost thermal conductivity is a function of the volumetric ratio of frost columns, β, frost surface temperature, T_s, plate temperature, T_p, and the way the temperature varies within the frost layer.

The dependence of effective frost thermal conductivity on the water vapor diffusion can be explained by the volumetric ratio of frost columns, β, which changes with time continuously due to sublimation of water vapor around the frost columns.

3.3. Radial growth of frost columns

Vapor diffusion through the void portions of the frost layer causes the radius of a column to grow with time. The diffusion rate of mass of water vapor through the frost layer is given by Eq. (6). This can be expressed in terms of the temperature by using Eqs. (9) and (10) as

$$\dot{m}_{\rm d} = -0.622 \times 10^{-2} D\rho_{\rm a}\frac{h_{\rm ig}}{RT^2}P_0\exp\left\{\frac{h_{\rm ig}}{R}\left(\frac{1}{T_0}-\frac{1}{T}\right)\right\}\frac{{\rm d}T}{{\rm d}y}. \tag{16}$$

The amount of mass which crosses the frost surface and diffuses through the void parts of the frost solidifies around the frost columns causing a horizontal growth with time. During this horizontal growth the radius of a frost column is assumed to be the same everywhere in the vertical direction (i.e. β is kept uniform along y). However, the local density of frost varies along the frost columns due to the dependence of ice crystal density with the formation temperature as given in Eq. (1). The overall density of the frost layer is an average density considering the void portions of the frost layer as well,

$$\rho_{\rm f} = \frac{\beta}{L_{\rm f}}\int_0^{L_{\rm f}} \rho_{\rm c}\,{\rm d}y. \tag{17}$$

The initial value of the volumetric ratio of frost columns, β_0, which is needed to start the calculations in this analysis is obtained from experimental results and depends on the experimental parameters such as $T_{\rm p}$, $T_{\rm s}$ and $\omega_{\rm a}$. A regression analysis based on the experimental results of Şahin [30] gives the following linear correlation of β_0 with these experimental parameters:

$$\beta_0 = -11.8916 + 0.01371T_{\rm p} + 0.03269T_{\rm a} - 112.677\omega_{\rm a}. \tag{18}$$

Regression analysis shows that the effect of Reynolds number on β_0 is negligible.

The variation of the volumetric ratio of frost columns with time is calculated using the rate of mass of vapor diffusing through the frost layer, $\dot{m}_{\rm d}$, and the frost column density, $\rho_{\rm c}$. During each time step in the numerical procedure, it is assumed that the amount of

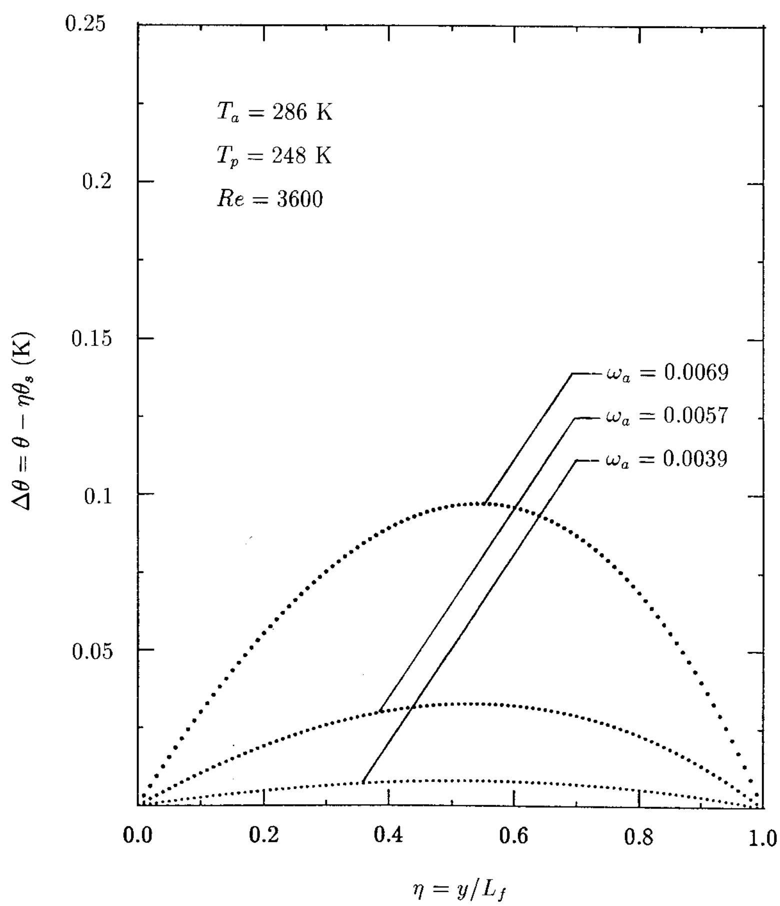

Fig. 7. Temperature variation inside the frost layer (t = 1 h).

mass of water vapor that diffuses inside the frost layer increases the radius of the frost columns uniformly in y direction, while the remaining mass of water vapor that arrives at the surface of the frost layer accumulates on the top of the frost columns, increasing their height. These changes in radius and height of frost columns are calculated considering the sublimation density of ice crystals given by Eq. (1) and evaluating it at the local temperature of the frost layer.

4. Results and discussion

The effective frost thermal conductivity, defined by Eq. (15), includes vapor diffusion and latent heat of sublimation which are two main processes taking place simultaneously during the frost formation. Although, the effective frost thermal conductivity expressions are suitable for analytical calculations, the above processes are treated separately in numerical analyses. Moreover, the frost formation process as a whole is transient in nature. However, the process of frost growth and densification is a very slow process. Therefore, the above analysis was carried out by a quasi-steady state assumption. Calculations of temperature variation, vapor diffusion, density and thermal conductivity were performed at every selected time interval, Δt, and the geometrical dimensions of frost columns were modified at the end of each Δt. A time interval of 1 min was selected in this study.

The temperature variation inside the frost layer and the surface temperature in particular is believed to be the key function for determining all of the frost formation parameters. The water molecules arriving at the cold surface of frost columns form ice crystals whose shape and density are strongly related to the local temperature at which they are formed. For this reason, one should pay more attention to the tempera-

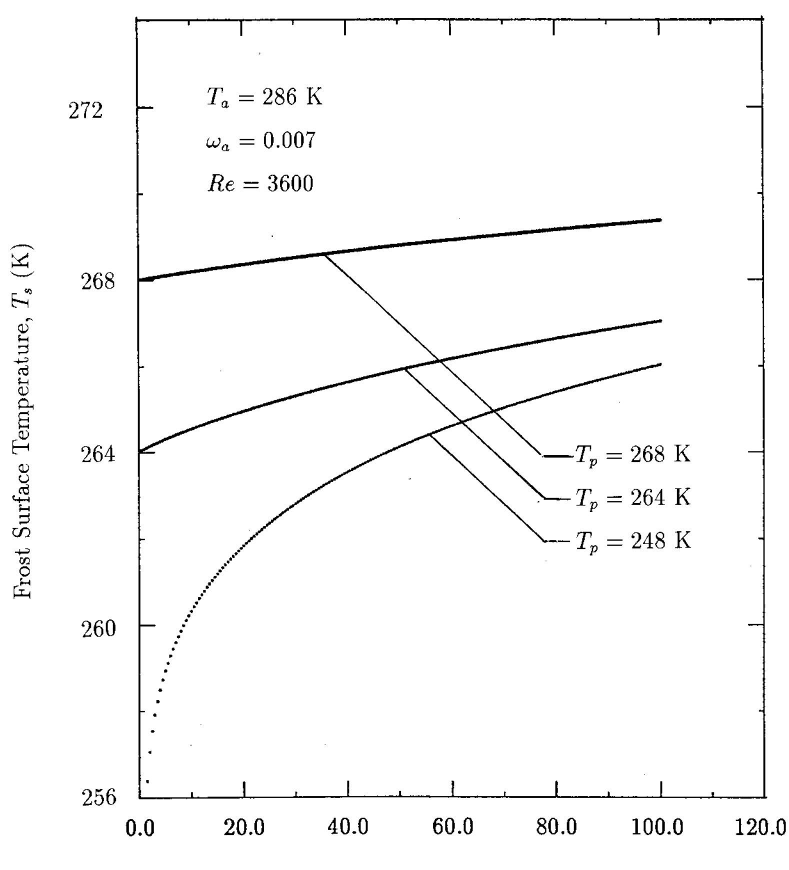

Fig. 8. Frost surface temperature variation with time.

ture variation in the frost layer and its variation with time.

Integrating Eq. (7) once

$$k_f(T)\frac{dT}{dy} = \frac{1}{L_f}\int_{T_p}^{T_s} k_f(T)\,dT$$

and integrating once more, the temperature variation in the frost layer can be obtained as

$$\frac{\int_{T_p}^{T} k_f(T)\,dT}{\int_{T_p}^{T_s} k_f(T)\,dT} = \frac{y}{L_f}.$$

The solution of this equation must be done numerically.

Starting with zero thickness of frost layer and suggested β_0 from Eq. (18), the initial amount of water vapor diffusion, $\dot{m}$, is calculated. Then the total frost deposited on the cold surface during the first time interval (Δt) is obtained. At the end of Δt, the density, the height and the thermal conductivity values of the frost columns are determined. Based on these values, the temperature variation along the frost columns and the surface temperature of the frost (top of columns) are calculated. All these calculations are repeated with the new frost surface temperature for the second time interval Δt. After each Δt, size of frost columns, density and effective thermal conductivity of the frost layer are recalculated, until the total time of 100 min is reached. In the numerical calculations, the thickness of the frost layer is divided into sublayers each one of which corresponds to one of the time intervals. As time progresses the number of sublayers increase. The thickness of sublayers, their density and thermal conductivity values are different from one another and are modified as time progresses. It should be noted that,

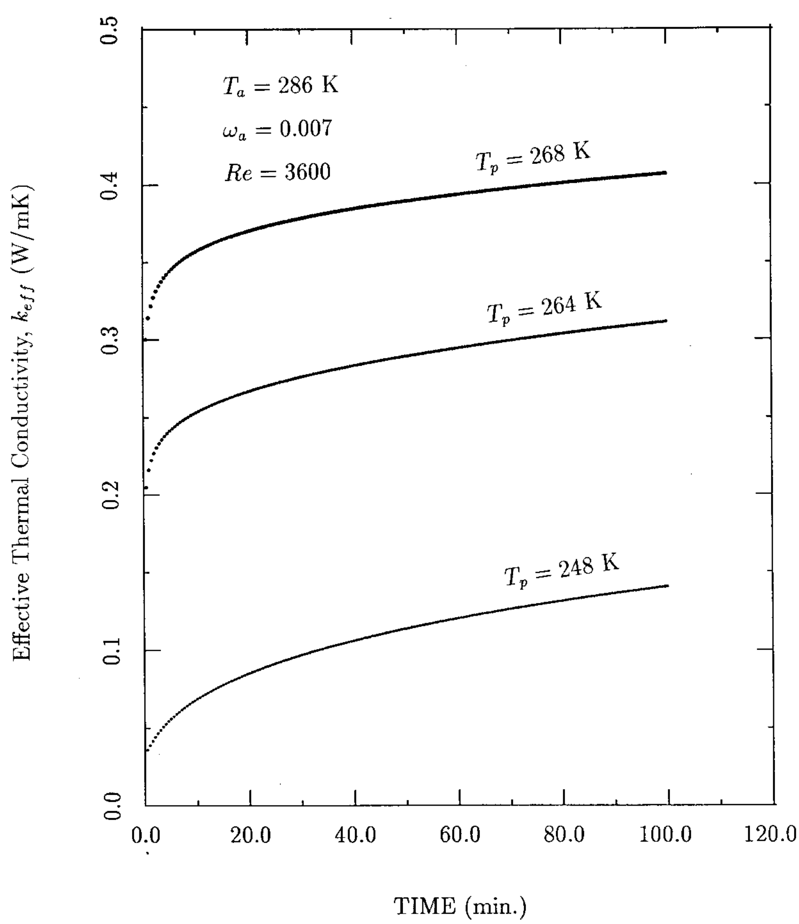

Fig. 9. The effect of plate temperature on effective frost thermal conductivity.

the present model may not give accurate results for the cases where frost temperatures are below 253 K since ice crystal densities below 253 K are not available and after times when the surface temperature of the frost reaches the melting temperature.

Fig. 7 shows a typical variation of temperature in the frost layer. The horizontal axis gives the dimensionless distance through the thickness of the frost layer while the vertical axis gives the temperature difference of the calculated temperature profile from the linear temperature distribution. θ in this figure is the temperature deviation from the plate temperature, $\theta = T - T_p$, and θ_s is the temperature difference between the surface temperature of the frost and the plate temperature, $\theta_s = T_s - T_p$. Thus, the term $(\theta - \eta\theta_s)$ represents the deviation of temperature from the linear variation. This deviation is mainly due to vapor diffusion and sublimation activity taking place in the frost layer.

The maximum deviation of the actual temperature distribution and the linear one with the same boundary temperatures, T_p and T_s, is found to be less than 0.5 K in most cases. Therefore, the temperature variation in the frost layer can be assumed to be linear at all times. It should also be noted that this small temperature deviation from the linear profile shows that the frost formation process is very slow and thus the quasi-steady state assumption is justified.

A typical variation of the surface temperature with time is given in Fig. 8. During the early stages of frost formation the surface temperature increases rapidly and then slows down approaching the melting point. The variation of the effective frost thermal conductivity with different frost parameters is given in Figs. 9–12. The effective frost thermal conductivity increases with time as a result of densification. It varies with variations in frost surface temperature, plate temperature and humidity ratio. In addition, the effect of

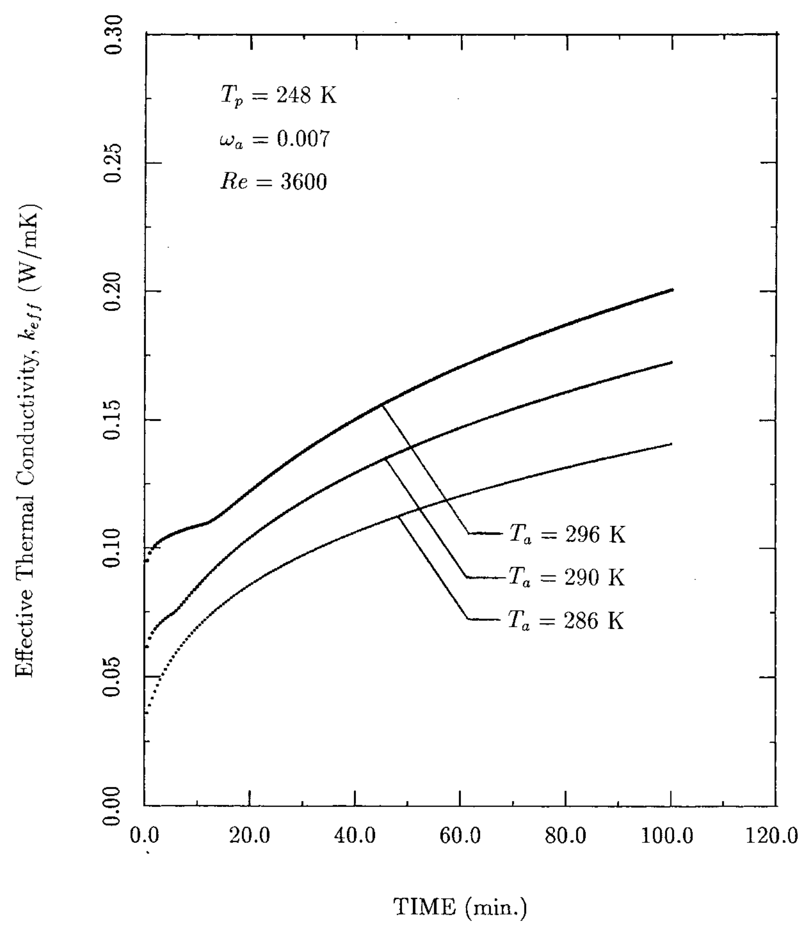

Fig. 10. The effect of air temperature on effective frost thermal conductivity.

water vapor which exists around the frost columns throughout the frost formation process is implemented during the development of effective frost thermal conductivity model. That is why the effective thermal conductivity increases rapidly in the early stages of the frost formation process, namely the crystal growth period, especially for the cases where driving potential for vapor diffusion is higher and then levels off approaching a steady value as the frost layer thickens. This shows that during the crystal growth period, frost thermal conductivity is not a function of only frost density. The structure of frost in this period is quite different than that of relatively thick frost layers. Therefore, the complex empirical correlations found in the literature related to thick frost layers and snow are not suitable for the crystal growth period of frost formation.

The effect of plate temperature on the effective frost thermal conductivity is considerable, as shown in Fig. 9. Higher plate temperature yields denser frost layer with high thermal conductivity. A similar result is obtained for the parameter of air temperature in Fig. 10, but the effect is smaller.

The effect of humidity ratio of air on the effective frost thermal conductivity is shown in Fig. 11. Higher humidity ratio results in low effective frost thermal conductivity. It should be noted that for low humidity ratio, the effective frost thermal conductivity stays nearly constant except during the very early period of frost formation (i.e, for the first 5–10 min with a humidity ratio of 0.003). This is because the frost layer is developing in a dense structure, such that the density is high and diffusion of vapor in the frost layer is almost negligible.

Fig. 12 shows the effect of Reynolds number on the effective frost thermal conductivity. Effective frost thermal conductivity is slightly greater for higher Reynolds numbers due to the dominant factor of densification.

Numerical results of the effective frost thermal con-

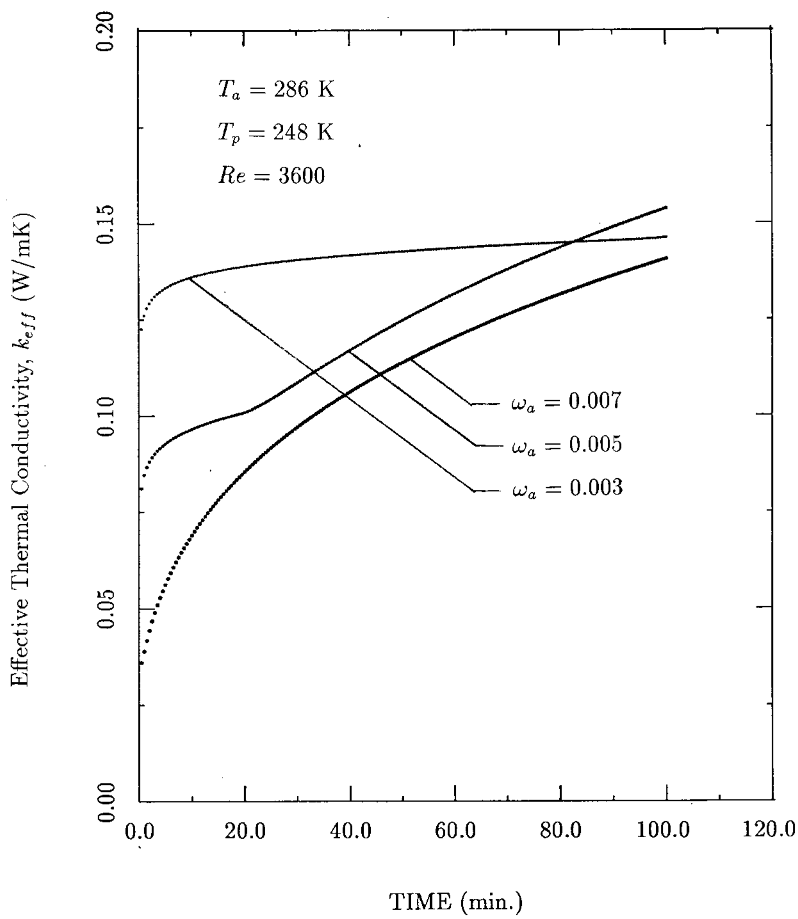

Fig. 11. The effect of air humidity ratio on effective frost thermal conductivity.

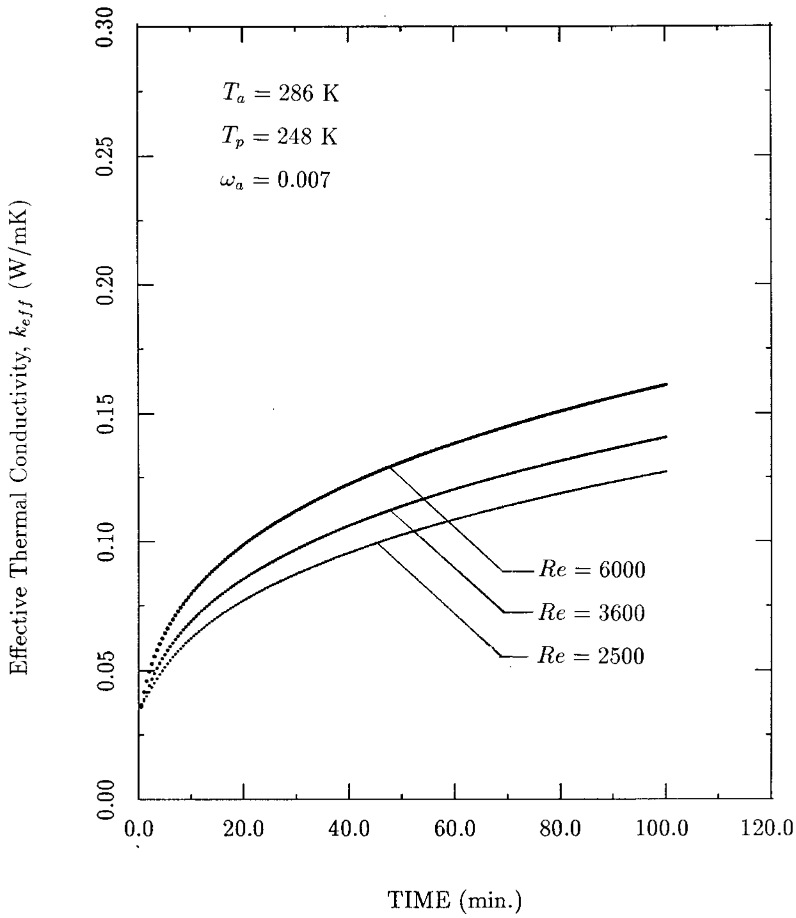

Fig. 12. The effect of Reynolds number on effective frost thermal conductivity.

ductivity and the average frost density for several sets of frost formation parameters are plotted on Fig. 13. The results do not show a functional dependency between the effective frost thermal conductivity and the density. The numerical values of the effective thermal conductivity are spread out in a region rather than forming a curve on this figure. This means that the effective frost thermal conductivity is not a function of only density, however the density can be regarded as the influencing parameter. One can always approximate these results with a single relationship. But there are cases where this assumption may lead to significant errors. Frost layers with a density of 200 kg/m^3 may have effective thermal conductivity values between 0.13 and 0.2 W/m K. This is a variation of more than $\pm 20\%$ based on the average effective thermal conductivity for the same frost density.

5. Conclusions

An effective frost thermal conductivity was introduced. The effect of water vapor that exists around the frost columns throughout frost formation process is implemented during the development of effective frost thermal conductivity model.

The effective frost thermal conductivity increases with time, in general, as a result of densification. Higher plate temperature results in more dense frost layer with high effective frost thermal conductivity. A similar result is seen for the parameter of air temperature, but the effect is smaller. High humidity ratio results in low effective frost thermal conductivity while low humidity ratio yields nearly constant but higher effective frost thermal conductivity due to negligible vapor diffusion in the frost layer. Frost thermal con-

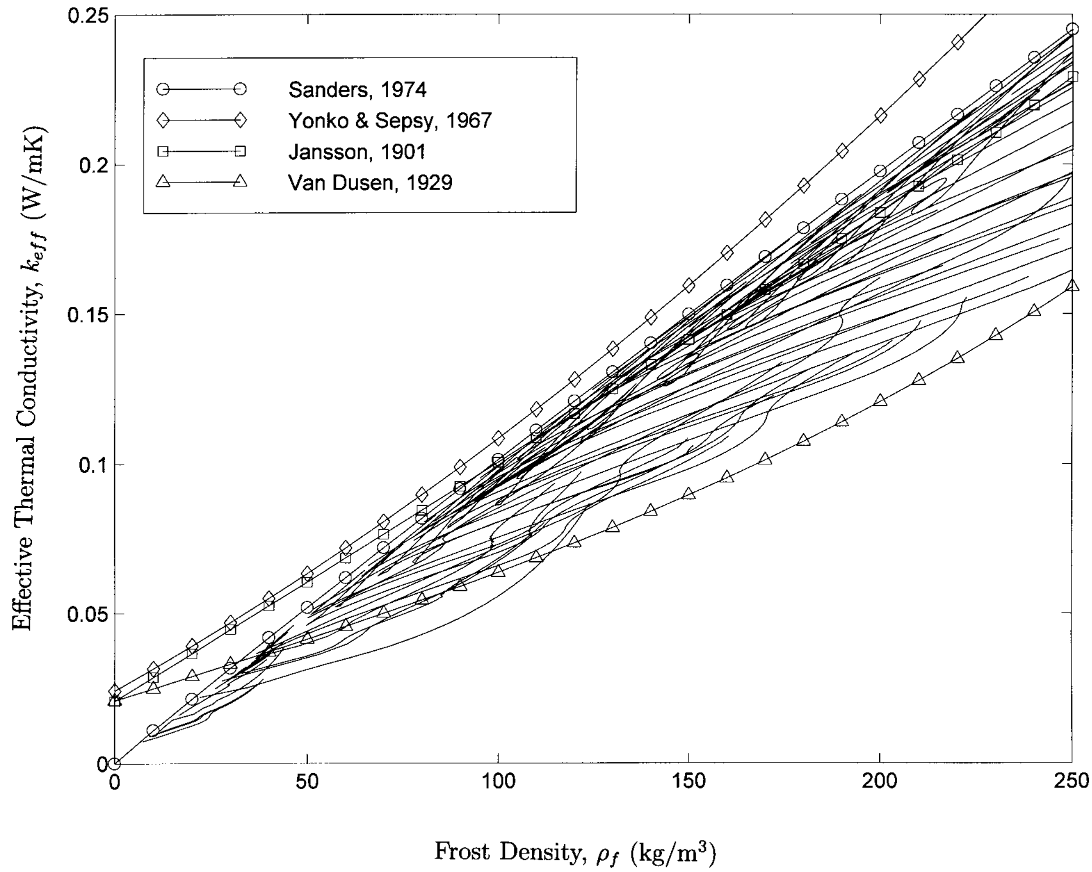

Fig. 13. Numerical results of effective frost thermal conductivity versus frost density for 100 different sets of frost parameters.

ductivity is slightly greater for higher Reynolds numbers due to the dominant factor of densification.

All the parameters affecting the frost density have influence on the effective frost thermal conductivity. But effective frost thermal conductivity cannot be related to frost density as a sole parameter.

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